

**MULTIPOLE MIXING RATIOS OF GAMMA
TRANSITIONS FROM LEVELS IN ^{23}Na
POPULATED IN THE $^{24}\text{Mg}(t,\alpha\gamma)^{23}\text{Na}$ REACTION**

FATIMA A. JASSIM

FIRAS M. HADY

Physics Dept., Education College, Ibn Al-Haitham,
Baghdad University.

ABSTRACT:

In the present study, the constant statistical tensor (CST) method has been used to calculate the multipole mixing ratios, δ -values of γ -transitions from excited levels in $^{24}\text{Mg}(t,\alpha\gamma)^{23}\text{Na}$ reaction.

The results obtained confirm the validity of this method in calculating the δ -values, CST-method also has been successfully used to calculate the δ -values of gamma transitions from excited levels of ^{23}Na with undefined parities.

INTRODUCTION:

The multipole mixing ratio (δ) is physical quantity which can be extracted from angular distribution measurements, in phase convention (δ) is defined as "the ratio of emission matrix elements" Steffan and Alder [1,2].

Multipole mixing ratio (δ) value can be used to help discern nuclear structure and transition properties, constant statistical tensor method one of important method can be used to calculate the δ -values for γ -transitions from excited levels, this method depend on that; the statistical tensor coefficient is the same for all levels with the same spin value depend neither upon energy level nor upon its parity, by using this fact in present work many transitions from excited levels with undefined parities were calculated.

The main aim of the present work was to confirm the validity of CST-method for calculating the δ -values for γ -transitions.

DATA REDUCTION AND ANALYSIS:

²⁴Mg(t,αγ)²³Na reaction for all possible γ-transitions, the multipole mixing ratios (δ) values can be calculated by using the following relationship;

$$a_2(J_i - J_f) = \rho_2(J_i) \frac{F_2(J_f L_1 L_1 J_i) + 2\delta F_2(J_f L_1 L_2 J_i) + \delta^2 F_2(J_f L_2 L_2 J_i)}{1 + \delta^2} \text{-----(1)}$$

a₂ – coefficient determined experimentally .

J_i = Spine of initial state.

J_f = Spine of final state.

ρ₂(J_i) = Statistical tensor coefficient depends only on the J_i –value.

L = Angular momentum of the γ-transition. **L₁=1** , **L₂= L₁+1** , **L ≠ 0**

F₂ coefficients have been taken from [3,4].

For pure transition **δ=0** then equation (1) becomes;

$$a_2(J_i - J_f) = \rho_2(J_i) F_2(J_f L_1 L_1 J_i) \text{-----(2)}$$

The statistical tensor can then calculated from the following equation:

$$\rho_2(J_i) = \frac{a_2(J_i - J_f)}{F_2(J_f L_1 L_1 J_i)} \text{-----(3)}$$

In the present work the γ-transitions , $(\frac{9^+}{2} - \frac{5^+}{2})$ can be considered pure E₂

$(\frac{3^+}{2} - \frac{3^-}{2})$, $(\frac{3^-}{2} - \frac{5^+}{2})$ are considered pure E₁ , while 2702 KeV

$(\frac{7^+}{2} - \frac{7^+}{2})$ transition from the 4780 KeV level can be considered pure M₁ as ref.[5] suggested.

For pure transitions from levels in ^{23}Na equation (2) becomes;

$$\rho_2\left(\frac{9}{2}\right) = \frac{a_2\left(\frac{9}{2} - \frac{5}{2}\right)}{F_2\left(\frac{5}{2} 22 \frac{9}{2}\right)} = -\frac{a_2\left(\frac{9}{2} - \frac{5}{2}\right)}{0.43252} \text{-----(4)}$$

$$\rho_2\left(\frac{7}{2}\right) = \frac{a_2\left(\frac{7}{2} - \frac{7}{2}\right)}{F_2\left(\frac{7}{2} 11 \frac{7}{2}\right)} = -\frac{a_2\left(\frac{7}{2} - \frac{7}{2}\right)}{0.43644} \text{-----(5)}$$

$$\rho_2\left(\frac{3}{2}\right) = \frac{a_2\left(\frac{3}{2} - \frac{3}{2}\right)}{F_2\left(\frac{3}{2} 11 \frac{3}{2}\right)} = -\frac{a_2\left(\frac{3}{2} - \frac{3}{2}\right)}{0.40000} \text{-----(6)}$$

$$\rho_2\left(\frac{3}{2}\right) = \frac{a_2\left(\frac{3}{2} - \frac{5}{2}\right)}{F_2\left(\frac{5}{2} 11 \frac{3}{2}\right)} = -\frac{a_2\left(\frac{3}{2} - \frac{5}{2}\right)}{0.10000} \text{-----(7)}$$

The experimental a_2 - coefficient reported in ref.[6] are using.

Since there is two values for $\rho_2\left(\frac{3}{2}\right)$ then the weighted average value is taken into consideration.

For initial state with spin $J_i = \frac{5}{2}$ there is no pure transition

Fig.(1) ref.[6] , so $\rho_2\left(\frac{5}{2}\right)$ value was calculated in present work as follow ; multipole mixing ratios of the values , -0.08(3) ref.[6] , -0.08(2) ref.[7] , 0.05(6) ref.[7] , -0.045(15) ref.[8] , -0.06(4) ref.[9] , for 440 KeV $\left(\frac{5}{2} - \frac{3}{2}\right)$ transition from the 440 KeV level were used by equation (1) to calculate $\rho_2\left(\frac{5}{2}\right)$ values , then the weighted average value of $\rho_2\left(\frac{5}{2}\right)$ values is taken in calculation.

The statistical tensors $\rho_2(J_i)$ thus calculated are considered to be constant for all levels with the same J_i – values , the values obtained for $\rho_2(J_i)$ can then be used in equation (1) to calculate the δ – mixing ratios for all γ -transitions whose a_2 – coefficients were reported in ref.[6].

For such transitions from levels in ^{23}Na equation(1) becomes;

$$a_2\left(\frac{5}{2} - \frac{3}{2}\right) = \rho_2\left(\frac{5}{2}\right) \frac{0.37417 - 1.89738\delta - 0.19090\delta^2}{1 + \delta^2} \text{----- (8)}$$

$$a_2\left(\frac{7}{2} - \frac{5}{2}\right) = \rho_2\left(\frac{7}{2}\right) \frac{0.32733 - 1.88984\delta - 0.07793\delta^2}{1 + \delta^2} \text{----- (9)}$$

$$a_2\left(\frac{9}{2} - \frac{5}{2}\right) = \rho_2\left(\frac{9}{2}\right) \frac{-0.43252 - 1.09108\delta - 0.41287\delta^2}{1 + \delta^2} \text{----- (10)}$$

$$a_2\left(\frac{9}{2} - \frac{7}{2}\right) = \rho_2\left(\frac{9}{2}\right) \frac{0.30277 - 1.87084\delta - 0.01966\delta^2}{1 + \delta^2} \text{----- (11)}$$

$$a_2\left(\frac{3}{2} - \frac{3}{2}\right) = \rho_2\left(\frac{3}{2}\right) \frac{-0.40000 - 1.5492\delta + 0.00000\delta^2}{1 + \delta^2} \text{-----} (12)$$

$$a_2\left(\frac{3}{2} - \frac{5}{2}\right) = \rho_2\left(\frac{3}{2}\right) \frac{0.10000 + 1.18322\delta + 0.35714\delta^2}{1 + \delta^2} \text{-----} (13)$$

$$a_2\left(\frac{3}{2} - \frac{1}{2}\right) = \rho_2\left(\frac{3}{2}\right) \frac{0.50000 - 1.73206\delta - 0.50000\delta^2}{1 + \delta^2} \text{-----} (14)$$

$$a_2\left(\frac{5}{2} - \frac{3}{2}\right) = \rho_2\left(\frac{5}{2}\right) \frac{0.37417 - 1.89738\delta - 0.19090\delta^2}{1 + \delta^2} \text{-----} (15)$$

$$a_2\left(\frac{5}{2} - \frac{7}{2}\right) = \rho_2\left(\frac{5}{2}\right) \frac{0.13363 - 1.38874\delta + 0.32453\delta^2}{1 + \delta^2} \text{-----} (16)$$

$$a_2\left(\frac{3}{2} - \frac{5}{2}\right) = \rho_2\left(\frac{3}{2}\right) \frac{0.10000 + 1.18322\delta + 0.35714\delta^2}{1 + \delta^2} \text{-----} (17)$$

$$a_2\left(\frac{3}{2} - \frac{7}{2}\right) = \rho_2\left(\frac{3}{2}\right) \frac{-0.14286 - 0.9258\delta + 0.50000\delta^2}{1 + \delta^2} \text{-----} (18)$$

$$a_2\left(\frac{3}{2} - \frac{5}{2}\right) = \rho_2\left(\frac{3}{2}\right) \frac{0.10000 + 1.18322\delta + 0.35714\delta^2}{1 + \delta^2} \text{-----} (19)$$

$$a_2\left(\frac{3}{2} - \frac{7}{2}\right) = \rho_2\left(\frac{3}{2}\right) \frac{-0.14286 - 0.9258\delta + 0.50000\delta^2}{1 + \delta^2} \text{-----} (20)$$

$$a_2\left(\frac{5}{2} - \frac{5}{2}\right) = \rho_2\left(\frac{5}{2}\right) \frac{-0.42762 - 1.01418\delta + 0.19090\delta^2}{1 + \delta^2} \text{-----} (21)$$

$$a_2\left(\frac{3}{2} - \frac{3}{2}\right) = \rho_2\left(\frac{3}{2}\right) \frac{-0.40000 - 1.5492\delta + 0.00000\delta^2}{1 + \delta^2} \text{-----} (22)$$

RESULTS AND DISCUSSION:

The statistical tensors , $\rho_2\left(\frac{3}{2}\right)$, $\rho_2\left(\frac{5}{2}\right)$, $\rho_2\left(\frac{7}{2}\right)$ and $\rho_2\left(\frac{9}{2}\right)$ calculated as mentioned in subsection using the experimental a_2 – coefficients reported in ref.[6] were as follows ;

$$\rho_2\left(\frac{3}{2}\right) = -1.08245 \pm 0.07261$$

$$\rho_2\left(\frac{5}{2}\right) = -0.80791 \pm 0.11017 \quad , \quad \rho_2\left(\frac{9}{2}\right) = -1.01770 \pm 0.071003$$

$$\rho_2\left(\frac{7}{2}\right) = -0.96233 \pm 0.11456$$

These values were used to calculate the δ -values of γ -transitions from excited levels of ^{23}Na isotope , by solving the equations that is related to gamma transitions , i.e. equations (8) to (22) .

The results thus obtained are presented and compared with those of ref.[5] , ref.[6]. in Table (1), it is clear from Table(1) that there is two δ -values for each γ -transition.

Small δ -values calculated for all γ -transitions are in good agreement with those reported in ref.[5] and ref. [6].

This indicates that a_2 - coefficients reported for these transitions are accurate.

The δ -value of 3302 KeV $\left(\frac{5^+}{2} - \frac{7^+}{2}\right)$ transition from the level 5380 KeV is in good agreement with thus of ref.[6] ,where there is discrepancy with the δ -value of ref.[5] , however the associated error of δ -value in ref.[5] is higher than the value it self , no further discussion can be made large δ -values calculated for γ -transitions have not been reported in ref.[5] , ref.[6].

Also δ -values for gamma transitions from excited levels with underline parities and not reported in ref.[6] have been calculated as in Table(1) (these transitions are not considered in ref.[5]) by using CST –method.

CONCLUSION:

The δ - mixing ratios of gamma transitions from ^{23}Na levels populated in the nuclear reaction $^{24}\text{Mg}(t,\alpha\gamma)^{23}\text{Na}$ have been calculated in the present work using CST –method.

The results obtained have confirmed not the validities of this method as a tool for calculating the δ -values of such transitions only but its capability to calculate the δ -values for γ -transitions from excited level whose parities undefined , by using the fact that , the statistical tensor coefficient is the same for all levels with the same spin value depend neither upon energy level nor upon its parity.

REFERENCES:

- 1-**Steffen R.M. and Alder K., (1975), Angular Distribution and Correlation of Gamma Rays-1. Theory, Edited by Hamilton W.D., pub. North Holland, Amsterdam, Chapter XII.
- 2-**Krane K.S., (1980), Atomic Data Nucl. Data Table, 25,29.
- 3-**Al-Ezzy F.A.A. , (2005), Study The Nuclear Structure of Some Deformed Nuclei , M.Sc. Thesis, Al-Mustansriyah University.
- 4-**Al-Zuhairy M.H.M., (2002), Multipole Mixing Ratios From The Heavy Ions Reaction By Using Constant Statistical Tensor Method , Ph.D. Thesis , Baghdad University.
- 5-**Forestone R.B. and Shirley V.S. , (1999), Table of Isotopes , 8th edition , John Wiley and Sons.
- 6-**Poletti A.R. , Becker J.A. and McDonald R.E., (1970), Phys. Rev. , C2 , p964.
- 7-**Poletti A.R. and Start D.F.H. , (1966), Phys. Rev. , 147 , p800.
- 8-** Mlzobuchi A. , Katoh T. and Ruan J. , (1960), Phys. Soc. Japan 15 , p1737.
- 9-**Sowerby B.D. , Sheppard D.N. and Olsen W.C. , (1968), Nucl.Phys. , A121 , p181.

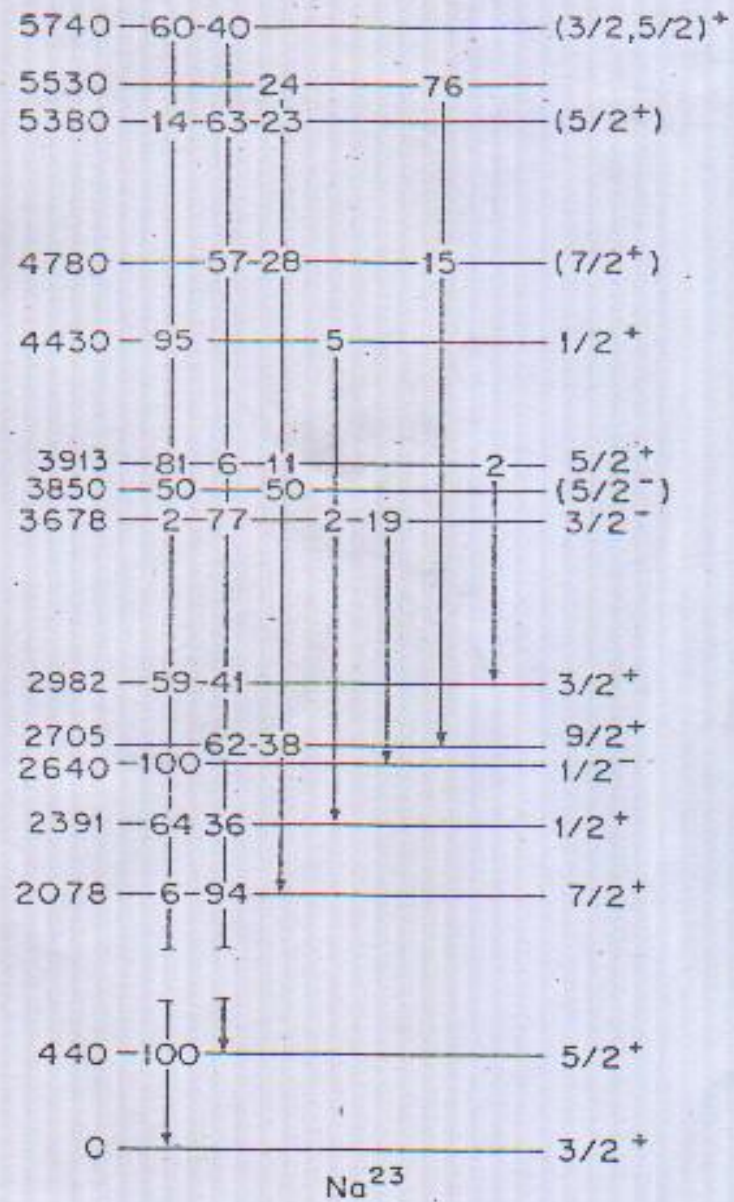


Fig.(1) : ^{23}Na levels populated in the nuclear reaction $^{24}\text{Mg}(t,\alpha)^{23}\text{Na}$. ref.[6]

Table(1):Multipole mixing ratios of gamma transitions from ^{23}Na levels calculated by constant statistical tensor (CST) method.

E_i (KeV)	E_γ (KeV)	$J_i^\pi - J_f^\pi$	a_2	$\delta - values$		
				CST method	Ref.[6]	Ref.[5]
440	440	$\frac{5^+}{2} - \frac{3^+}{2}$	$-(0.22 \pm 0.03)$	0.053(3)	-0.08(3)	-0.058(3)
2078	1638	$\frac{7^+}{2} - \frac{5^+}{2}$	0.01 ± 0.03	0.18(1) $-(28.2)_{-5}^{+24}$	-0.18(4) -----	0.19 (2) -----
2705	2265 627	$\frac{9^+}{2} - \frac{5^+}{2}$ $\frac{9^+}{2} - \frac{7^+}{2}$	0.43 ± 0.08 $-(0.30 \pm 0.06)$	<i>E2</i> 0.00(4)	0.01(3) -0.04(3)	<i>E2</i> 0.08(2)
2982	2982 2542	$\frac{3^+}{2} - \frac{3^-}{2}$ $\frac{3^+}{2} - \frac{5^+}{2}$	0.44 ± 0.03 $-(0.03 \pm 0.07)$	0.00(3) -0.06(6) $-(3.5)_{-0.6}^{+1}$	-0.03(6) 0.07(21) -----	-0.01(2) 0.09(3) -----
3678	3238 1038	$\frac{3^-}{2} - \frac{5^+}{2}$ $\frac{3^+}{2} - \frac{1^-}{2}$	$-(0.08 \pm 0.03)$ $-(0.65 \pm 0.06)$	-0.02(1) -0.07(5) -1.5(2)	0.01(5) 0.11(6) -----	-0.02(3) -0.13(5) -----
3913	3913 1835	$\frac{5^+}{2} - \frac{3^+}{2}$ $\frac{5^+}{2} - \frac{7^+}{2}$	0.07 ± 0.02 $-(0.32 \pm 0.08)$	-0.25(3) $-(17.9)_{-3.3}^{+7}$ 0.19(9) 19.2(10)	-0.22(3) -0.12(12) -----	0.22(3) 0.12(12) -----
4780	4340	$\frac{3}{2} - \frac{5}{2}$ $\frac{5}{2} - \frac{5}{2}$ $\frac{7^+}{2} - \frac{5^+}{2}$	$-(0.05 \pm 0.02)$	-0.05(3) -3.8(2) -0.44(4) 8.3(2) 0.14(2) $-(14.7)_{-2.1}^{+2.9}$	----- ----- ----- ----- -0.15(4) -----	----- ----- ----- ----- 0.17(2) -----

Table(1): Continued.

E_i (KeV)	E_γ (KeV)	$J_i^\pi - J_f^\pi$	a_2	$\delta - values$			
				<i>CST method</i>	Ref.[6]	Ref.[5]	
4780	2702	$\frac{7^+}{2} - \frac{7^+}{2}$	0.42 ± 0.05	M1	0.06(12)	-0.04(10)	
5380	4940	$\frac{3}{2} - \frac{5}{2}$	0.24 ± 0.03	-0.32(5)	-----	-0.16(6)	
	3302	$\frac{5^+}{2} - \frac{5^+}{2}$		-1.7(2)	-----		
$\frac{5^+}{2} - \frac{5^+}{2}$		-0.12(5)	0.16(7)	-----			
$\frac{7}{2} - \frac{5}{2}$		2.2(3)	-----	-----			
$\frac{7}{2} - \frac{5}{2}$		0.31(2)	-----	-----			
			$(10.7)^{+3.7}_{-2.2}$	-----			
	3302	$\frac{3}{2} - \frac{7}{2}$	0.05 ± 0.07	-0.10(6)	-----	0.04(9)	
		$\frac{3}{2} - \frac{7}{2}$		-1.8(3)	-----		
		$\frac{5^+}{2} - \frac{7^+}{2}$		-0.15(7)	-0.19(12)		-----
		$\frac{5^+}{2} - \frac{7^+}{2}$		$-(3.4)^{+1.2}_{-0.7}$	-----		
		$\frac{7}{2} - \frac{7}{2}$	-0.43(3)	-----			
			2.9(3)	-----			
5740	5740	$\frac{3}{2} - \frac{3}{2}$	0.20 ± 0.09	-0.14(5)	-----	+0.17(3)	
		$\frac{5^+}{2} - \frac{3^+}{2}$		$(8.5)^{+7.1}_{-2.7}$	-----		
		$\frac{5^+}{2} - \frac{3^+}{2}$		0.33(7)	-0.30(14)		-----
				33.2(230)	-----		

5300	$\frac{3}{2} - \frac{5}{2}$	0.21 ± 0.04	$-0.29(5)$ $-1.9(2)$ $-0.15(5)$ $2.4(4)$	----- ----- 0.19(12) -----	0.19(12) -----
------	-----------------------------	-----------------	---	-------------------------------------	-------------------

نسب خلط متعدد الأقطاب لانتقالات أشعة كاما
من مستويات في ^{23}Na متولدة في التفاعل
 $^{24}\text{Mg}(t,\alpha\gamma)^{23}\text{Na}$

فراس محمود هادي

فاطمة عبد الأمير جاسم

قسم الفيزياء / كلية التربية - ابن الهيثم / جامعة بغداد

الخلاصة:

لقد تم في هذا البحث حساب نسب الخلط (δ) للانتقالات الكامية من المستويات المتهيجة في التفاعل النووي $^{24}\text{Mg}(t,\alpha\gamma)^{23}\text{Na}$ باستعمال طريقة التنسر الإحصائي الثابت ، وقد أثبت الاتفاق الجيد بين قيم (δ) المحسوبة بهذه الطريقة والقيم المستخدمة في البحث (5) والبحث (6) لانتقالات أشعة كاما كما ليس في صحة طريقة التنسر الإحصائي الثابت في حساب نسب الخلط (δ) فقط وإنما استعمالها بنجاح أداة" لحساب نسب الخلط لانتقالات كاما لم يتم تعيينها من مستويات مجهولة التماثلية في البحث (6).