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New class of connected topological spaces

By

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المستخلص :-

تم في هذا البحث دراسة خصائص لصنف أكثر توسعا" من الفضاء المترابط وهو الفضاء (perfect connected) وكذلك تم دراسة التطبيق (magic-almost open) وتمت البرهنة على إن كل تطبيق حقيقي معرف على فضاء perfect connected يكون تطبيق Magic-almost open

وكذلك تم تعريف التطبيق المفتوح تقريبا" والاستفادة منه في البرهنة على إن كل تطبيق حقيقي معرف على فضاء (perfect connected) يكون مستمر تطبيق تشاكل هوميومورفزم .

Abstract :-

In this paper we study the properties a large class of connected topological spaces : the class is perfect connected spaces. Also we study the magic-almost open map and we prove every real continuous map defined on a perfect connected spaces then its magic- almost open map. Also we defined the almost- open map [1] and we used to prove for every real continuous map defined on a perfect connected space then the map is homomorphism.

Definition (1):-A topological space (X,T) is said to be perfect connected space if there exist a connected and locally connected space Y and a continuous onto map f: X → Y.

Remark (1):-Let Z be non empty subset of the topological space X we say that the set Z is perfect connected space if Z with relative topology is perfect connected set.

Example (1) :-The usual topological space (R, U) is perfect connected space since the topological space (R, U) is connected and locally connected space and we can defined the identity map $I:R\to R$ which is onto continuous map.

Definition (2) :-Let X and y be two- topological space the onto map $f:X\to Y$ is said to be almost open map iff for all point y Є Y. There exists a point $x\in f(y)$ such that f is interior map in a point x. [3]

Lemma (1) :-The map $f:X\to Y$ is almost open map iff for all y Є Y then $f^{-1}(y)\cap \text{int} f\neq \emptyset$.

<u>Example : (2)</u> :-Let X = {a, b, c} and T_x = {ø, x,{a}, {c}, {a,c}} be the topological defined on X and let Y={x,y} and T_y = {ø , y , {x}} be the topological defined on y .
Let f: X→ Y be the map definition by :f (b) = f (c) =y Э f (a) = x
Since f^{-1} (y) ∩ int f ≠ Ø $\forall\ y \in Y$ then by the lemma (1-1-8) f is almost open map.

<u>Remark: (2)</u>:-If {Xi} is a family of subsets of perfect connected space on the topological space X and $\bigcap_{i \in I} X_i \neq \phi$ then $\bigcup_{i \in I} X_i$ is completely perfect set.

<u>Remark (3)</u>:-
1) Every perfect connected space is connected space.
2) Every connected space and locally connected is completely perfect space.

<u>Definition (3)</u>:-Let f: X→R be a map defined on the topological space X then f is called magic-almost open map if every point t Є f (x) there exist a non empty subset Kt ⊆ f^{-1} (t) (Card Kt ≤ 2) such that every neighborhood Kt ⊆ U then f (a) is neber hood the point t.

<u>Remark (4)</u>:-The subset A of the topological space X is open and closed set iff

$$b(A)=\phi$$

<u>Theorem (1)</u>:- Let (X,T) be a topological space and X^* be a connected subset on X and let the points x_0 , x_1 belong to X^* the map g: X →R definition on X then:

I- x0 is local max point (local min point) on the map g.
II- g (x_0) < g (x_1).
III- g is continuous map for every point on X^* then there exist a point $x^* \in X^*$ Satisfy the following:

1- g (X^*) = g (x_0) .

2- X^* Is not local max point (local min) for a map g

3- x^* is not local min point (local max) for a map g if for all open set $\Omega \subseteq X$,$\Omega \cap X^*$ there exist the point $\bar{x} \in \Omega$ such as that $g(\bar{x}) \neq g(x_\circ)$

Proof: - Since the map has a local max in the point x_0, $\exists$ the neighborhood V for point x_0 such that $g(x) \leq g(x_0)$ $\forall x \in V$ .Let F be the set family of neighborhood which satisfy $g(x) \leq g(x_0)$, since V Є F then F ≠Ø. let $V^* = \bigcup V$ so that V^* Є F and since x_0 Є V^*, x_0 Є X^* then x_0 Є $V^* \cap X^*$ and $V^* \cap X^* \neq$ Ø . x_1 Є X^* and x_1 Є V^* so that X^*-$V^* \neq$ Ø and X^* is subset connected space then X^* is the only open and closed set , hence $b(V^* \cap X^*) \neq$ Ø, Now if $b(V^* \cap X^*)$ = Ø then $V^* \cap X^*$ is open and closed set (2-1-3) and there exist subset $V^* \cap X^*$ from X^* is open and closed set which is contradiction .

Let x^* Є $b(V^* \cap X^*)$ and let U be neighborhood for the point x^* hence $U \cap (V^* \cap X^*) \neq$Ø and $U \cap (X^* - V^*) \neq$Ø.

Proof (1): Let K = {x Є X^* :$g(x) \leq g(x_0)$ } because $g(x) \leq g(x_0)$ is satisfy for all neighborhood contained the point x^* and g is continuous map then $g(x^*) \leq g(x_0)$. assume $g(x^*) < g(x_0)$ and Let W be a neighborhood contained the point x^* and $W^* = V^* \cap W$ Since $V^* = \cap$ V and F be a family of all open neighborhood which satisfy $g(x) \leq g(x_0)$ So that W^* Є F and there for $W^* \leq V^*$. For all neighborhood N to x^* contains all points from $V^* \cap X^*$ and the point $\bar{X}^*$ -V^*, W is contain at least a point y from $V^* \cap X^*$ and a point $\bar{y}$ from

X^*- V^*, So that $\bar{y}$ Є $(X^* - V^*) \cap$ w and $\bar{y}$ Є $(X^* - V^*)$, $\bar{y}$ Є W. since

$W \subseteq V^*$ then $y \notin W$ that is contradiction. Hence $g(x^*) = g(x_0)$.

Proof (2): Assume x^* is a local min point for a map g then $\exists$ a neighborhood contains a point x^* such that $g(x) \geq g(x^*)$ $\forall x \in S$ since $g(x^*) = g(x_0)$ then $g(x) \leq g(x_0)$ $\forall x \in S$ and $S \subseteq V^*$ because

$\forall N$ for x^* contains the points for $V^* \cap X^*$ and point y_1, from X^* - V^* we get $y_1 \in (x^* - V^*) \bigcap S$ and $y_1 \in (X^* - V^*)$ and $y_1 \in S$ so that $y_1 \notin S$ is contradiction .Hence X^* is not local max point for map g.

Proof (3): Assume X^* is local minim point for a map g then a neighborhood S' contains point x^* such that g (x) ≥ g (x^*). $\forall x \in S'$ Let Ω = V^* ∩ S' then $\Omega \bigcap X^* \neq \phi$ by assume $\exists \bar{x} \in \Omega \ni g(\bar{x}) \neq g(x_o)$ because $x \in S'$ then $g(\bar{x}) > g(x_o)$ and $\bar{x} \in V^*$ then $g(\bar{x}) \leq g(x_o)$ that is contradiction so that x^* is not local min point for a map g by the same method if x is local min point for a map g and g (x_0) > g (x_1).

<u>Theorem (2)</u>:- Every real continuous map defined on perfect connected space then it's magic – almost open map.

Proof: - Let X is perfect connected space and f: X→ R be continuous real map and T= f (x) assume T= {t} now to prove f is magic – almost open map. Since

f (x) ={t} then $f^{-1}(t)$ = X and $\exists$ a non empty subset Kt $\subseteq$ X ∋ card Kt ≤ 2 So that $\exists$ a non empty subset Kt $\subseteq$ $f^{-1}(t)$ ∋ card Kt ≤ 2, then for all neighborhood Kt $\subseteq$ U, f(u) is a neighborhood for a point t. now assume that the interval T is not vanishing since X is perfect connected space then $\exists$ the space Y connected and locally connected and the map h: Y → Y is continuous map.

Let g =foh and t Є T. Assume inf (x) < t < sup f (x) ,by theorem (2-1-4) y^* Є Y sub such that g(y^*) = t and y^* is not locally max point for a map g and $\exists$ y^* Є Y such that g(y^*)= y and y^* is locally min point for a map g let u^* be a neighborhood contains the point y^* since y is Perfect connected

Then $\exists$ a connected neighborhood $V^* \ni y^* \in V^* \subseteq U^*$ and $t \in g(V^*) \subseteq g(U^*)$,$[t, t+\in] \subseteq g(U^*)$ $\forall \in > 0$ Because Y is perfect connected space then for all neighborhood U^{**} is contained the point y^* and $\exists$ connected a neighborhood $V^{**} \ni y^{**} \in V^{**} \subseteq U^{**}$ .then $t \in g(V^{**}) \subseteq g(U^{**})$,

$[t-\in, t] \subseteq g(U^{**})$ $\forall \in > 0$ and $\forall$ neighborhood N contained the set {

y*, y**}. Then g(N) is neighborhood contained the point t. let Kt ={h(y*) , h(y**) } and W be a neighborhood contained the set Kt since h is continuous map then ∃ an open neighborhood W_1 contained the point y* and h (w_1) ⊆ W also ∃ open neighborhood W_2 contained the point Y** and h (W_2) ⊆ W let U =W_1 U W_2 then h (u) = h (W_1) U h (W_2) and h (u) ⊆ W therefore f (h (u)) ⊆ f (w) i.e. g(u) ⊆ f(w) and hence f is magic – almost open map .
now assume t= inf (x) by theorem (2-1-4) ∃ y* Є Y Э g (y*) =t and y* is not locally max point for map g let Kt = {h (y*)} and W is a neighborhood contained the set Kt because h is continuous map then ∃ an open neighborhood W1 contained the point y* and f (h(w_1)) ⊆ f (w) i.e. g(w1) ⊆ f (w) there fore f is magic –almost open map by same method we prove t=sup f (X) .

Remark (4):-When card Kt = 1 the magic –Almost open map is almost open map.

Theorem (3):-Let X is a perfect connected space and f: X→ R be a one to one continuous map then f is homomorphism.

Proof: we must to prove f^{-1} is continuous map i.e. to prove f is open map. because f is continuous map and X is perfect connected by theorem (2-1-5) f is magic–Almost open map, and since f is one to one map then ∀ t Є f (X) , f^{-1}(t) is only one element that is mean card Kt=1 ∀ t Є f (X) by Remark (2-1-2) f is almost open map f^{-1} (t)⋂ in t f ≠ Ø ∀ t Є f (x) and f^{-1} (t) ⊆ int f also ∀ open set U in X Эf^{-1} (t) Є U , t is interior point in f (u) i.e. ∃ open set G Э t Є G ⊆ f (u) ∀ t Є f (X) is interior map in f^{-1} (t) then f is interior map for every point of X i.e. in f =X hence f is open map there for f is homomorphism .

Remark (5):- In the next example we prove if f: X→R is magic –Almost open map then f it's not necessary X is perfect connected.

Example (3):- let X be accountable connected hausdorff space open set A which homeomorphism with the rational number space. It is easy to show that every continuous map f: X→R is constant, so that f is magic –Almost open map and to show that let $t \in f(x)$, since f is constant map then $f^{-1}(t) = X$. then there exit anon empty subset $K_t \subseteq X \ni card \ K_t \leq 2$ and anon empty subset $K_t \subseteq f^{-1}(t) \ni card \ K_t \leq 2$ such that for each neighborhood $K_t \subseteq U$ then f(u) is neighborhood of the point t. and hence f is magic –Almost open map.

Now, if we suppose X is perfect connected space ,then by definition (2-1-1) there exist a connected and locally connected space Y with the continuous on to map $f : X \to Y$, implies that g(Y) =X. let z be the point in A and $B = g^{-1}(Z)$. Because X is hausdorff space then{z} is closed space ,and since g is continuous map then $B = g^{-1}(Z)$ is closed set in Y. let $b \in B$ because A is open set and g is continuous map then $\exists$ an open set U contained the point b such that $g(U) \subseteq A$,since V is connected space and g is continuous map then $V \subset g^{-1}(z)$ and $b \in V \subseteq B = g^{-1}(z)$. Hence B is connected that is contradiction.

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