Diala , Jour , Volume , 36 , 2009

المجموعات المفتوحة قبليا القوية والدوال المستمرة قبليا القوية
في
الفضاءات التبولوجية المضببة الحدسية الخاصة

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الاختصاص / تبولوجي مضبب

المستخلص

إن الهدف من هذا البحث هو تعميم مفاهيم المجموعات المفتوحة قبليا القوية المضببة في المصدر [1] الى الفضاءات التبولوجية المضببة الحدسية الخاصة وعلاقتها بالمجموعات المفتوحة والمجموعات شبه المفتوحة القوية . وقد دُرست بعض القواعد والخواص ،كذلك استخدمنا هذه المفاهيم في بحث عدة نظريات مكافئة للدوال المستمرة قبليا القوية في الفضاءات التبولوجية المضببة الحدسية الخاصة .

1429 2008

STRONGLY PREOPEN SETS AND STRONG PRECONTINUITY IN INTUITIONISTIC FUZZY SPECIAL TOPOLOGICAL SPACES

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ABSTRACT

In this paper, the concepts of fuzzy strongly preopen (resp. fuzzy strongly preclosed) of fuzzy set due to (B.Krsteska)[1], have been generalized to intuitionistic fuzzy special topological spaces and some of its basic properties are studied .using these concepts we investigate several characterizing theorem of intuitionistic fuzzy special strongly precontinuous funtions

1. Introduction

After the introduction of the concept of fuzzy sets by Zadeh [6] several reseaches were conducted on the generalizations of the notion of fuzzy set. The idea of intuitionistic fuzzy set was frist published by krassimir Atanassov [6] .Coker [5] defined the intuitionistic fuzzy topological spaces . The concepts is used to define intuitionistic fuzzy special sets by Coker [6] and intuitionistic fuzzy special topological spaces are introduced.

In section 3 we introduce intuitionistic fuzzy special strongly preopen sets and we study some of their properties . We discuss the relationships between this class and classes defined previously .Also we introduce the strong preinterior and strong preclosure of intuitionistic fuzzy special set. In section 4 we introduce intuitionistic fuzzy special strongly precontinuous .Also we produce characterizing theorem of intuitionistic fuzzy special strongly precontinuous.

2. Preliminaries

Now we introduce some basic notions and results that are used in the sequel.

***Definition 2.1*[7]**

Let X be a nonempty set .An intuitionistic fuzzy special set A is an object having the form $A = < x \;, \; A_1, A_2 >$ *, where* A_1 *and* A_2 *are subsets of X satisfying* $A_1 \cap A_2 = \phi$ *.The set* A_1 *is called the set of members of A , while* A_2 *is called the set of nonmembers of A*

***Definition2.2* [7]**

An intuitionistic fuzzy special topology on a nonempty set X is family T of intuitionistic fuzzy special sets in X satisfying the following conditions.

1. $\tilde{\Phi} \;, \tilde{X} \in T$.
2. *T is closed under finite intersection.*
3. *T is closed under arbitrary unions.*

The pair (X, T) is called an intuitionistic fuzzy special topological space and any intuitionistic fuzzy special set in T known open set in X.

From now the word space means an intuitionistic fuzzy special topological space.

The complement of an open set in a space (X, T) is called closed set.

Definition2.3

Let (X,T) be a space and let A be an intuitionistic fuzzy special set of X . Then A is called :

1. *An intuitionistic fuzzy special semi-open set (SOS, for short) iff $A \subseteq cl\,(int(A))$ [2]*
2. *An intuitionistic fuzzy special preopen set (POS, for short) iff $A \subseteq in(cl(A))$ [2].*

3. *An intuitionistic fuzzy special strongly semiopen set(SSOS, for short) if and only if $A \subseteq int(cl(int(A)))$. [3]*

Definition 2.4 *[3]*

Let (X,T) be a space and let A be an intuitionistic fuzzy special set of X . Then A is called :

1. *An intuitionistic fuzzy special semiclosed set (SCS for short) if and only if A^c is semiopen set of X .*
2. *An intuitionistic fuzzy special preclosed set (PCS for short) if and only if A^c is preopen set of X .*
3. *An intuitionistic fuzzy special strongly semiclosed set (SSCS for short) if and only if A^c is strongly semiopen set of X.*

Definition 2.5 *[3,2]*

Let A be an intuitionistic fuzzy special set of a space (X, T) , then

1. SSintA = $\cup\{ B : B \subseteq A$, B is strongly semiopen set of X $\}$ is called intuitionistic fuzzy special strongly semi-interior of A

2. SSCl=$\cap\{ B : A \subseteq B$, B is strongly semiclosed set of X $\}$ is called Intuitionistic fuzzy special strongly semiclosure of A .

3.pint A =$\cup\{B: B \subseteq A$, B is preopen set of X$\}$ is called an intuitionistic fuzzy special preinterior of A.

4.pcl A = $\cap\{ B : B \supseteq A$, B is preclosed set of X$\}$ is called intuitionistic fuzzy special preclosure of A

Theorem2.6 [6]

Let A be an intuitionistic fuzzy special set of a space (X, T) , then

1. $pcl = A \cup cl\ (intA)$,
2. $pint\ A = A \cap int(cl\ A\ )$.

Lemma 2.7 *[1]*

Let A be an intuitionistic fuzzy special set of a space (X, T) , then

1. $pcl\ A^c = (\ pint\ A)^c$
2. $pint\ A^c = (\ pcl\ A\ )^c$

Lemma 2.8 *[4]*

Let $f : X \to Y$ *be a function ,for an intuitionistic fuzzy special set A and B of X and Y respectively , the following statements hold :*

1. $f\ f^{-1}(B) \subseteq B$, *if f is surjective, then* $f\ f^{-1}(B) = B$.
2. $A \subseteq f^{-1}f\ (A)$ *and if f is injective then ,* $f^{-1}f\ (A) = A$.
3. $f^{-1}(B^c) = f^{-1}(B)^c$
4. $f\ (A)^c \subseteq f\ (A^c)$ *and if f is bijective ,then* $f\ (A^c) = f\ (A)^c$

Definition 2.9

Let $f : X \to Y$ *be a function from a space (X,T) into a space* (Y,∂) *,then:*

1. *f is continuous function if* $f^{-1}(B)$ *is open set of X ,for each open set B in Y . [4]*
2. *f is precontinuous if* $f^{-1}(B)$ *is pre-open set of X for each open set B in Y . [3]*
3. *f is open (closed) function if f(A) is open (closed)set of Y for each open set A in X [3]*
4. *f is pre-open (pre-closed) function if f (A) is pre-open (pre-closed) set of Y for each open set A in X [4]*

3. Intuitionistic fuzzy special strongly preopen sets and intuitionistic fuzzy special strongly preclosed sets

Definition3.1

Let (X,T) be a space and let A be an intuitionistic fuzzy special set of X . Then A is called strongly preopen set if and only if $A \subseteq int\,(pclA)$

The family of all intuitionistic fuzzy special strongly preopen sets of a space (X,T) will denoted by SPO(X), the complement of strongly preopen set A^c is called strongly perclosed set and the family of all strongly preclosed set denoted by SPC(X).

Lemma3.2

Let A be an intuitionistic fuzzy special set of a space (X, T) , the following properties hold :

1. $int\,(pclA) \subseteq int\,(clA)$;
2. $int(pclA) \supseteq int\,(cl(intA))$.

Proof

1. *It follows from the relation $pcl\,A \subseteq cl\,A$,for any intuitionistic fuzzy special set A of X .*
2. *fromTheorem[2.6],we have $int(pclA) \supseteq int(A \cup (cl(intA)) \supseteq int(cl(intA))$.*

Theorem3.3

Let (X,T) be any intuitionistic fuzzy special topological space, then the following statements hold :

1. *Every open set is strongly preopen set.*
2. *Every strongly semiopen set is strongly preopen set .*
3. *Every strongly preopen set is preopen set .*

proof

It follows easily from lemma [3.2]

The following examples shows that inverse of above theorem is not true in general.

Example3.4

Let X = { a,b,c} , T = { $\tilde{\Phi}$, $\tilde{X}$, A ,B } where A= < x , {a}, {b,c }> B= < x, {a,b} , ϕ > ,let C= < x, {a }, {b }> , C is not open set but C is strongly preopen set since int(pclA) =X and C$\subseteq$X.

Example3.5

Let X = { a,b,c } , T= { $\tilde{\Phi}$,$\tilde{X}$,A ,B } where A=< x, {a},{b} > , B = < x , {a,b} , ϕ >

Let C= < x, {b,c}, {a} > , C is preopen set since intcl A=X ,but C is not strongly preopen set since int(pcl A)= ϕ and C $\not\subset$ ϕ .

Theorem3.6

Let A be an intuitionistic fuzzy special set of a space (X, T) , then A is intuitionistic fuzzy special strongly preclosed set if and only if cl(pintA)$\subseteq$A.

Proof

Let A be any intuitionistic fuzzy special strongly preclosed set ,then A^c is strongly preopen set .

From the relation $A^c \subseteq$ int (pclA^c) we have cl(pintA)$\subseteq$A.

Conversely

Let A be an intuitionistic fuzzy special set such that cl(pintA)$\subseteq$A ,from the relation

$A^c \subseteq$int(pclA^c) ,it follows that $A^c$is strongly preopen set .

Hence A is strongly preclosed set .

Lemma3.7

Let $\{A_{\propto}\}_{\propto\in J}$, be a family of an intuitionistic fuzzy special sets of a space (X, T) ,then $\cup_{\propto\in J}\ pcl(A_{\propto}) \subseteq pcl\ (\cup_{\propto\in J} A_{\propto})$

Theorem3.8

1. *Any union of intuitionistic fuzzy special strongly preopen sets is intuitionistic fuzzy special strongly preopen set*
3. *Any intersection of intuitionistic fuzzy special strongly preclosed sets is intuitionistic fuzzy special strongly preclosed set .*

Proof

1. *Let $\{A_{\propto}\}_{\propto\in J}$ be any family of intuitionistic fuzzy special strongly preopen sets .For each $\propto\in J$, $A_{\propto} \subseteq int(pcl\ A_{\propto})$.hence from lemma[3.7] We have $\cup_{\propto\in J} A_{\propto} \subseteq \cup_{\propto\in J}\ int(pcl\ A_{\propto}) \subseteq int\ (pcl \cup_{\propto\in J} A_{\propto})$.*
2. *Let $\{A_{\propto}\}_{\propto\in J}$ be any family of intuitionistic fuzzy special strongly preoclosed sets .Thus $\{A^{c}_{\propto}\}_{\propto\in J}$ be family of intuitionistic fuzzy special strongly preopen sets . According to (1) $\cup_{\propto\in J} A^{c}_{\propto}$ is intuitionistic fuzzy special strongly preopen sets.from $(\cup_{\propto\in J} A^{c}_{\propto})^{c} = \cap_{\propto\in J} A_{\propto}$ we obtain the conclusion.*

Definition 3.9

Let A be an intuitionistic fuzzy special set of a space (X, T) ,then

1. *spint A = $\cup$\{ B : B is intuitionistic fuzzy special strongly preopen set of X, and $B \subseteq A$\} is called intuitionistic fuzzy special strongly preinterior of A.*
2. *spcl A = $\cap$\{ B: B is intuitionistic fuzzy special strongly preoclosed set ,and $A \subseteq B$ \} is called intuitionistic fuzzy special strongly preclosure of A .*

proposition3.10

Let A be an intuitionistic fuzzy special set of a space (X, T) ,then

1. *int A $\subseteq$ spint A $\subseteq$ A ; A $\subseteq$ spcl A $\subseteq$ cl A*
2. *spint A $\in$ intuitionistic fuzzy special SPO(X) ; spcl A $\in$ intuitionistic fuzzy special SPC(X)*
3. *A $\in$ intuitionistic fuzzy special SPO(X) $\Leftrightarrow$ A = spint A ; A $\in$ intuitionistic fuzzy special SPC(X) $\Leftrightarrow$ A = spcl A*
4. *A $\subseteq$ B $\Rightarrow$ spint A $\subseteq$ spintB ; A $\subseteq$ B $\Rightarrow$ spcl A $\subseteq$ spcl B*
5. *spint (spint A)= spint A ; spcl (spcl A)= spcl A*

6. $spint\ A \cap spint\ B \supseteq spint\ (A \cap B)$; $spcl\ A \cup spcl\ B \subseteq spcl\ (A \cup B)$
7. $spint\ A \cup spint\ B \subseteq spint\ (A \cup B)$; $spcl\ (A \cap B) \subseteq spcl\ A \cap spcl\ B$
8 . $spintX = X$; $spcl\phi = \phi$.

Proof

It follows from def [3.9], and theorem [3.8]

The following theorem gives a characterization of strongly preopen (strongly preclosed) sets .

Theorem3.11

1. *An intuitionistic fuzzy special set B of a space (X,T) is strongly preopen set if only if there exists intuitionistic fuzzy special set A of X such that* $A \subseteq B \subseteq int\ (\ pcl\ A)$
2. *An intuitionistic fuzzy special set B of a space (X,T) is strongly preclosed set if and only if there exists intuitionistic fuzzy special set A of X such that* $cl(pintA) \subseteq B \subseteq A$

Proof *we prove only the statement (1)*

Let B be an intuitionistic fuzzy special of X , if A an intuitionistic fuzzy special of X such that $A \subseteq B \subseteq int\ (pclA)$*exists , then* $B \subseteq int(pclA) \subseteq int(pclB)$*. Thus B is intuitionistic fuzzy special strongly preopen.*

Conversely

If B is any intuitionistic fuzzy special strongly preopen.

, then the result follows for A = B .

In same way we can prove (2) by taking the complement .

Theorem 3.12

Let A be an intuitionistic fuzzy special set of a space (X, T) ,then

1. $spcl\ A^c = (\ spint\ A)^c$.
2. $spint\ A^c = (\ spcl\ A)^c$.

proof

1. *by theorem [3.10](1)* $spint\ A \subseteq A$ *and* $spint\ A \in SPO(X)$ *,then* $A^c \subseteq (spint\ A)^c$*and* $(spint\ A)^c$ *is strongly preclosed set in X*

Hence $spcl\ A^c \subseteq (spintA)^c$ --------(1)

Conversely

By theorem [3.10] $A^c \subseteq (spcl\ A^c)$ *and* $(spcl\ A^c)$ *strongly preclosed set in X*

Then $(splcA^c)^c \subseteq A$ *and* $(spcl\ A^c) \in SPO(X)$

So that $(spclA^c)^c \subseteq spint\ A$

Hence $(spint\ A)^c \subseteq spcl\ A^c$ *--------- (2)*

From (1) & (2) we get $(spint\ A)^c = spclA^c$.

2. $(spclA)^c = (spcl(A^c)^c)^c = ((spintA^c)^c)^c = spint\ A^c$.

***Theorem*3.13**

Let A be an intuitionistic fuzzy special set of a space (X, T) ,then

$intA \subseteq ssintA \subseteq spintA \subseteq pintA \subseteq A \subseteq pclA \subseteq spclA \subseteq ssclA \subseteq clA$

proof

It follows from the definitions of corresponding operators .

4. *Intuitionistic fuzzy special strong precontinuity*

***Definition* 4.1**

A function $f : X \to Y$ *from a space (X,T) into a space(* Y, ∂ *) is called intuitionistic fuzzy special strongly precontinuous if* $f^{-1}(B)$ *is strongly preopen set in X ,for each open set B in Y .*

***Theorem*4.2**

Let $f : X \to Y$ *be a function from a space (X,T) into a space(* Y, ∂ *),then the following statements are equivalent .*

1. *f is intuitionistic fuzzy special strongly precontinuous function .*
2. $cl(pint\ f^{-1}(B)) \subseteq f^{-1}(clB)$ *, for each intuitionistic fuzzy special set B of Y.*
3. $f(cl(pintA)) \subseteq cl\ f(A)$*, for each intuitionistic fuzzy special set A of X.*

Proof

The proof is standard and therefore omitted .

Theorem4.3

Let $f : X \to Y$ *be a function from a space (X,T) into a space(* Y, ∂ *),then the following statements are equivalent .*

1. *f is intuitionistic fuzzy special strongly precontinuous function;*
2. $f^{-1}(B)$*is intuitionistic fuzzy special strongly preclosed set of X , for each intuitionistic fuzzy special closed set B of Y ;*

3. $f(spclA) \subseteq cl\, f(A)$,*for each intuitionistic fuzzy special set A of X;*
4. $spcl\, f^{-1}(B) \subseteq f^{-1}(clB)$, *for each intuitionistic fuzzy special set B of Y ;*
5. $f^{-1}(intB) \subseteq spint(f^{-1}(B))$,*for each intuitionistic fuzzy special set B of Y .*

Proof

$1 \to 2$

Let B any closed set in Y , then B^c *is open set in Y , since f is strongly precontinuous then* $f^{-1}(B^c)$ *is strongly preopen set in X ,*
But $f^{-1}(B^c) = (f^{-1}B)^c$*.lemma[2.8](3) then* $f^{-1}(B)$ *is strongly preclosed set in X.*

$2 \to 3$

Let A be an intuitionistic fuzzy special set of X ,then cl(f(A))is closed set of Y, by (2) $f^{-1}(cl(f(A)))$*is strongly preclosed set of X and*
$spcl\, A \subseteq spcl((f^{-1}(f(A))) \subseteq spcl(f^{-1}(clf(A))) = f^{-1}(cl(f(A)))$
Thus $f(spclA) \subseteq f f^{-1}(cl(f(A)) \subseteq cl\, f(A)$.

$3 \to 4$

Let B be any intuitionistic fuzzy special set of Y by (3) $f(spcl(f^{-1}(B))) \subseteq cl\,(f f^{-1}(B)) \subseteq cl\,(B)$.
Thus $spcl(f^{-1}(B) \subseteq f^{-1}(f(spcl(f^{-1}(B)))) \subseteq f^{-1}(clB)$.

$4 \to 5$

Let B an intuitionistic fuzzy special set of Y by (4)
$f^{-1}(clB^c) \supseteq spclf^{-1}(B^c) = (spcl\, f^{-1}(B))^c$
From theorem [3.12], we have
$f^{1}(intB) = f^{-1}(clB^c)^c = (f^{-1}(clB)^c)^c \subseteq (spcl\, f^{-1}(B^c))^c = spint(f^{-1}(B))$.

$5 \to 1$

Let B open set in Y, then B=intB
By (5) $f^{-1}(B) = f^{-1}(intB) \subseteq spint(f^{-1}(B)) \subseteq f^{-1}(B)$
,i.e. $f^{-1}(B) = spint(f^{-1}(B))$
From prop. [3.10], we have $f^{-1}(B)$ *is strongly preopen set in X*
Thus f is strongly precontinuous .

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